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Robustness of the Mean Chart to Non-Normality and Measurement Error

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Abstract: In this paper we discussed the robustness of the mean chart to non-normality and measurement error with known *cv*. It is seen that many areas of statistical application can easily lead to quite non-normal distribution as well as erroneous observation. The joint effect of non-normality and measurement error on the OC and type-I error has been investigated. The non-normal distribution has been represented by the first four terms of an Edgeworth series visual comparison shows that the effect of non-normality and measurement error is serious on the OC function and type-I error.

Key words: Mean Chart, OC function, type-I error, non-normality, Measurement error

INTRODUCTION

A problem faced in the context of control charts generally is the measurement error variability. This problem is the result of the inability to measure accurately the variable of interest. The use of imprecise measurement devices affects the ability of the control chart to detect on out-of-control situation. Moreover, the variable under interest may be related through a covariate with the measurement system used.

Mitkag and Stemann (1998) examined the effect of measurement error on the joined $\overline{X}, S$ control chart assuming the model of the form x = x + e, where x is the actual value of the variable and x is the measured value because of the random error. Linna and Woodall (2001) extended the preceding model by assuming one with covariates and they investigated the effect of this model in the $\overline{X}$ and S^2 control charts. Linna et al. (2001) examined the effect of the model with covariates in the case of a multivariate Showhart chart for mean. In a control chart we have two objectives. First, when we are in control, we want our chart to signal (false alarm) as we have planned it to do. In statistical terms, we want the chart to operate with the planned probability of the mean plotting outside the control limits if we are in control. Secondary, when the control chart is out of control, we want it to signal as soon as possible. In statistical terms we want the probability of the mean plotting in control if we are out of control to be as small as possible. Cornell (1995) defined a mixture experiment as one in which the response variable depends only on the proportion of the components or ingredients present the mixture and not on their amounts. Because sum of the component proportions present in any mixture sum to a constant, the levels of these variables not be set independently. Typically, there are also additional restrictions on the maximum and / or mimum proportions of some components. In many dustrial settings, mixture experiments also involve process variables. These are factors, that can be adjusted independently of one another and mixture components. Mixture-process variable experiments are discussed by, Piepel and Cornell (1985, 1987) and Myers and Montgomery (2002).

An important variation of the mixture process variable experiment is the case where one more of the process variables are either uncontrollable or difficult to control. This type variable is usually called a noise variable, and while it is uncontrollable or difficult to control in operation, it usually can be controlled during an experiment. The experiment is usually interested in determining the levels of the mixtures components and the controllable process variable that (i) optimize the mean response and (ii) minimize the variability transmitted into the response from the noise variables. This is one version of the robust design problem originally introduced by Taguch (1986, 1987) Modern approach to this problem are based on response surface methodology, for example Myers et al. (1992, 2004), Lucas (1994), and Myers and Montgomery (2002).

For more discussion of these idea, including the role of the slope, Myers and Montgomery (2002) and Borror et al. (2002). Goldfarb et al. (2004) provided graphical methods for evaluating designs for mixture process variable experiments with control and noise variables. Goldfarb et al. (2005) show how designs for mixture - process variable experiments with control and noise variables can be constructed.

In this paper we discussed the robustness of the mean chart to non-normality and measurement error with known *cv*. It is seen that many areas of statistical application can easily lead to quite non-normal distribution as well as erroneous observation. The joint effect of non-normality and measurement error on the OC and type-I error has been investigated. The non-normal distribution has been represented by the first four terms of an Edgeworth series visual comparison shows that the effect of non-normality and measurement error is serious on the OC function and type-I error.

SAMPLING DISTRIBUTION OF OBSERVED MEAN TO NON-NORMALITY AND MEASUREMENT ERROR

Assuming that the true measurement X and the random error of measurement e are additive, we can write the observed measurement X as :

$$X = x + e, \tag{1}$$

where x and e are independent. The constant μ and σ_e are the mean and stardard deviation of the true quality measurement x. As usual if we take e ~ N (0, σ_e^2) and independent of x, the correlation coefficient ρ between the true and the observed measurement can be found out to be

$$\rho = \sigma_p / \sigma_x , \tag{2}$$

where σ_x is the standard deviation of X. Since x and e are independent, the r^th^ cumulant of X (= x + e) is equal to the sum of the r^th^ cumulants of x and e. Further, since e ~ N (0, σ_e^2), all the cumulants of X and x respectively, we have

$$k_r = l_r , \qquad r \neq 2$$

$$k_2 = l_2 ,$$

Let v_r and λ_r $(r \neq 2)$ be the r^th^ standardized cumulants of X and x respectively. Then

$$v_r = \frac{k_r}{k_2^{\,r/2}} = \frac{l_r}{\sigma_x^{\,r}} = \frac{l_r}{\left(\sigma_{p/\rho}\right)^r}$$

$$\text{or} \qquad v_r = \rho^r \ \lambda_r \tag{3}$$

Following Srivastava and Banarasi (1980) the $MSE\left(\bar{x}^*\right) = MSE\left(\bar{x}^*\right) = \frac{\sigma^2}{n}\left(1 - \frac{v * \lambda_{ap}(\alpha_1, \alpha_2, n)}{n}\right)$ under measurement error.

The control chart for mean is set up by drawing the control line at the process average μ and the control limits at $\mu \pm \left(k\sigma / \sqrt{n}\right)$, where σ is the process standard deviation and n is the sample size. The OC function gives the probability that the control chart indicate the process average as μ when it is actually not μ but $\mu' = \mu + \gamma \frac{\sigma\sqrt{(1 - v / n * \rho)}}{\sqrt{n}}$ (say) and it is derive by integrating the distribution of mean with μ' as the process average between the limits of the control chart.

In case of known *cv* the non-normal population is represented by the first four terms of an Edgeworth series by Rao and Bhatt (1989) as,

$$f(x) = \frac{1}{\sigma}\left\{\left(\frac{x-\mu}{\sigma}\right) - \frac{\lambda_3}{6n}\phi^{(3)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_4}{24n}\phi^{(4)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_3^2}{72n}\phi^{(6)}\left(\frac{x-\mu}{\sigma}\right)\right\} \tag{4}$$

Following the $MSE\left(\bar{x}^*\right)$ the distribution of sample mean $\bar{x}^*$ is given by Gayen (1949), as

$$g\left(\bar{x}^*\right) = \frac{\sqrt{n'}}{\sigma}\left\{\phi\left(\frac{\bar{x}*-\mu}{\sigma/\sqrt{n'}}\right) - \frac{\lambda_3}{6\sqrt{n'}}\phi^{(3)}\left(\frac{\bar{x}*-\mu}{\sigma/\sqrt{n'}}\right) + \frac{\lambda_4}{24n'}\phi^{(4)}\left(\frac{\bar{x}*-\mu}{\sigma/\sqrt{n'}}\right) + \frac{\lambda_3^2}{72n'}\phi^{(6)}\left(\frac{\bar{x}*-\mu}{\sigma/\sqrt{n'}}\right)\right\} \tag{5}$$

where $M^2 = \left(1 - \frac{v}{n * \rho}\right), \ n' = \frac{M^2}{n}, \ \phi(x) = \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$ and

$$\phi^{(r)}(x) = \left(\frac{d}{dx}\right)^r \phi(x).$$

The OC function is obtained, after replacing μ in equation (5) by μ' and integrating it between the limits of control chart ϕ as,

$$L' = L_N - L'_u + L'_b, \tag{6}$$

where $$L_N = \Phi\left(\frac{k}{M}+\gamma\right) + \Phi\left(\frac{k}{M}-\gamma\right) - 1,$$

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)\,dt$$

L_N is the OC function the control chart when the underlying population is normal with known *cv*.

$$L'_u = \frac{n'}{72}\left[\frac{12\lambda_3}{\sqrt{n'}}\phi^{(2)}\left(\frac{k}{M}-\gamma\right) - 3\lambda_4\phi^{(3)}\left(\frac{k}{M}-\gamma\right) - \lambda_3^2\phi^{(5)}\left(\frac{k}{M}-\gamma\right)\right] \tag{7}$$

$$L'_b = \frac{n'}{72}\left[\frac{12\lambda_3}{\sqrt{n'}}\phi^{(2)}\left(\frac{k}{M}+\gamma\right) + 3\lambda_4\phi_4^{(3)}\left(\frac{k}{M}+\gamma\right) + \lambda_3^2\phi^{(5)}\left(\frac{k}{M}+\gamma\right)\right] \tag{8}$$

$$\alpha' = 1 - \int_{\mu-k\sigma/\sqrt{n}}^{\mu+k\sigma/\sqrt{n}} g(\bar{x}^*)\, d\bar{x}^*$$

$$= \alpha_N - \alpha_C \tag{9}$$

where $$\alpha_N = 2\Phi\left(-\frac{k}{M}\right) \tag{10}$$

and $$\alpha_c = \frac{n'}{36}\left[3\lambda_4\phi^{(3)}\left(\frac{k}{M}\right) + \lambda_3^2\phi^{(5)}\left(\frac{k}{M}\right)\right], \tag{11}$$

is the non - normality correction for the error of the first kind.

If α'_u is the probabilty of exceeding, the upper control limit and α'_b, that lying below the lower control limit, when the process is in control, then positively skewed population $\alpha'_u > \alpha'_b$ and for negatively skewed populations $\alpha'_u < \alpha'_b$. The expressions for α'_u and α'_b are given as

$$\alpha'_u = \frac{\alpha}{2} + \frac{n'}{72}\left\{12\lambda_3\sqrt{n'}\,\phi^{(2)}\left(\frac{k}{M}\right) - 3\lambda_4\phi^{(3)}\left(\frac{k}{M}\right) - \lambda_3^2\phi^{(5)}\left(\frac{k}{M}\right)\right\} \tag{12}$$

$$\alpha'_b = \frac{\alpha}{2} + \frac{n'}{72}\left\{12\lambda_3\sqrt{n'}\,\phi^{(2)}\left(\frac{k}{M}\right) + 3\lambda_4\phi^{(3)}\left(\frac{k}{M}\right) + \lambda_3^2\phi^{(5)}\left(\frac{k}{M}\right)\right\} \tag{13}$$

TABULATION AND DISCUSSION OF RESULT

For various non-normal population with non-normality parameter (λ_3, λ_4) = (-0.5, 0), (-0.5, 0.5), (-0.5, 1.0), (-0.5, 2.0), (0, 0.5), (0, 1.0) and (0, 2.0); different error sizes r = ∞, 2, 4, 6 ; *v* = 0, 2, 4, and 6 the values of α' are given in Table (1) for k = 2, 3 and n = 7, 10 and 15. The values of the OC function for k = 3, n = 7 r = ∞, 2, 4, 6 and 0, 2, 4, 6 are tabulated in Table (2). The OC for negative values of λ_3 are not given separately, as they can be obtained from the OC values for the corresponding positive values.

If $v = 0$, r = ∞ then from the tables OC function is not much affected by standardized cumulants. As the size of the measurement increases the values of OC function in ereas according to their Skewness and Kurtosis. It may also be inferred that due to the kurtosis of the underlying population and increasing values of r, the control chart indicates the presence of assignable causes more often than is expected when no such causes are present. The effect on the error of the first kind increases with increasing values of r, k and also as the departure from normality increases.

Table-1: Type one Error for Mean under Measurement Error with Known cv

r=∞ ρ=1												
v			k=2					k=3				
	n	λ_4 λ_3	**α**	0	0.5	1	2	**α**	0	0.5	1	2
0	7	0	**0.0042**	0.0455	0.0461	0.0468	0.0481	**0.0027**	0.0027	0.0032	0.0036	0.0046
		±0.5	**0.0042**	0.0445	0.0452	0.0458	0.0471	**0.0027**	0.0028	0.0033	0.0037	0.0047
	10	0	**0.0126**	0.0455	0.0460	0.0464	0.0473	**0.0027**	0.0027	0.0030	0.0034	0.0040
		±0.5	**0.0126**	0.0448	0.0453	0.0457	0.0466	**0.0027**	0.0028	0.0031	0.0034	0.0041
	15	0	**0.0219**	0.0455	0.0458	0.0461	0.0467	**0.0027**	0.0027	0.0029	0.0031	0.0036
		±0.5	**0.0219**	0.0451	0.0454	0.0457	0.0462	**0.0027**	0.0027	0.0030	0.0032	0.0036
2	7	0	**0.0205**	0.0205	0.0211	0.0218	0.0231	**0.0005**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0205**	0.0200	0.0207	0.0213	0.0226	**0.0005**	0.0028	0.0029	0.0031	0.0033
	10	0	**0.0273**	0.0273	0.0278	0.0283	0.0293	**0.0009**	0.0027	0.0028	0.0030	0.0033
		±0.5	**0.0273**	0.0269	0.0274	0.0279	0.0288	**0.0009**	0.0028	0.0029	0.0031	0.0034
	15	0	**0.0331**	0.0331	0.0334	0.0337	0.0344	**0.0014**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0331**	0.0327	0.0331	0.0334	0.0341	**0.0014**	0.0028	0.0029	0.0030	0.0033
4	7	0	**0.0042**	0.0042	0.0045	0.0048	0.0054	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0042**	0.0042	0.0045	0.0048	0.0054	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0126**	0.0126	0.0130	0.0133	0.0141	**0.0002**	0.0027	0.0027	0.0028	0.0029
		±0.5	**0.0126**	0.0124	0.0128	0.0132	0.0139	**0.0002**	0.0027	0.0028	0.0028	0.0029
	15	0	**0.0219**	0.0219	0.0222	0.0225	0.0232	**0.0006**	0.0027	0.0028	0.0028	0.0030
		±0.5	**0.0219**	0.0217	0.0220	0.0223	0.0229	**0.0006**	0.0027	0.0028	0.0029	0.0030
6	7	0	**0.0000**	0.0000	0.0000	0.0000	0.0001	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0000**	0.0000	0.0001	0.0001	0.0001	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0033**	0.0033	0.0035	0.0036	0.0040	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0033**	0.0033	0.0035	0.0037	0.0040	**0.0000**	0.0027	0.0027	0.0027	0.0027
	15	0	**0.0126**	0.0126	0.0128	0.0131	0.0136	**0.0002**	0.0027	0.0027	0.0028	0.0028
		±0.5	**0.0126**	0.0125	0.0127	0.0130	0.0135	**0.0002**	0.0027	0.0028	0.0028	0.0028

Continued…

r=2 ρ=0.8944												
v			k=2					k=3				
	n	λ_4 λ_3	**α**	0	0.5	1	2	**α**	0	0.5	1	2
0	7	0	**0.0455**	0.0455	0.0461	0.0468	0.0481	**0.0027**	0.0027	0.0032	0.0036	0.0046
		±0.5	**0.0455**	0.0445	0.0452	0.0458	0.0471	**0.0027**	0.0028	0.0033	0.0037	0.0047
	10	0	**0.0455**	0.0455	0.0460	0.0464	0.0473	**0.0027**	0.0027	0.0030	0.0034	0.0040
		±0.5	**0.0455**	0.0448	0.0453	0.0457	0.0466	**0.0027**	0.0028	0.0031	0.0034	0.0041
	15	0	**0.0455**	0.0455	0.0458	0.0461	0.0467	**0.0027**	0.0027	0.0029	0.0031	0.0036
		±0.5	**0.0455**	0.0451	0.0454	0.0457	0.0462	**0.0027**	0.0027	0.0030	0.0032	0.0036
2	7	0	**0.0205**	0.0205	0.0211	0.0218	0.0231	**0.0005**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0205**	0.0200	0.0207	0.0213	0.0226	**0.0005**	0.0028	0.0029	0.0031	0.0033
	10	0	**0.0273**	0.0273	0.0278	0.0283	0.0293	**0.0009**	0.0027	0.0028	0.0030	0.0033
		±0.5	**0.0273**	0.0269	0.0274	0.0279	0.0288	**0.0009**	0.0028	0.0029	0.0031	0.0034
	15	0	**0.0331**	0.0331	0.0334	0.0337	0.0344	**0.0014**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0331**	0.0327	0.0331	0.0334	0.0341	**0.0014**	0.0028	0.0029	0.0030	0.0033
4	7	0	**0.0042**	0.0042	0.0045	0.0048	0.0054	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0042**	0.0042	0.0045	0.0048	0.0054	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0126**	0.0126	0.0130	0.0133	0.0141	**0.0002**	0.0027	0.0027	0.0028	0.0029
		±0.5	**0.0126**	0.0124	0.0128	0.0132	0.0139	**0.0002**	0.0027	0.0028	0.0028	0.0029
	15	0	**0.0219**	0.0219	0.0222	0.0225	0.0232	**0.0006**	0.0027	0.0028	0.0028	0.0030
		±0.5	**0.0219**	0.0217	0.0220	0.0223	0.0229	**0.0006**	0.0027	0.0028	0.0029	0.0030
6	7	0	**0.0000**	0.0000	0.0000	0.0000	0.0001	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0000**	0.0000	0.0001	0.0001	0.0001	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0033**	0.0033	0.0035	0.0036	0.0040	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0033**	0.0033	0.0035	0.0037	0.0040	**0.0000**	0.0027	0.0027	0.0027	0.0027
	15	0	**0.0126**	0.0126	0.0128	0.0131	0.0136	**0.0002**	0.0027	0.0027	0.0028	0.0028
		±0.5	**0.0126**	0.0125	0.0127	0.0130	0.0135	**0.0002**	0.0027	0.0028	0.0028	0.0028

Continued…

r=4 ρ=0.9701												
v			k=2					k=3				
	n	λ_4 λ_3	**α**	0	0.5	1	2	**α**	0	0.5	1	2
0	7	0	**0.0455**	0.0455	0.0461	0.0468	0.0481	**0.0027**	0.0027	0.0032	0.0036	0.0046
		±0.5	**0.0455**	0.0445	0.0452	0.0458	0.0471	**0.0027**	0.0028	0.0033	0.0037	0.0047
	10	0	**0.0455**	0.0455	0.0460	0.0464	0.0473	**0.0027**	0.0027	0.0030	0.0034	0.0040
		±0.5	**0.0455**	0.0448	0.0453	0.0457	0.0466	**0.0027**	0.0028	0.0031	0.0034	0.0041
	15	0	**0.0455**	0.0455	0.0458	0.0461	0.0467	**0.0027**	0.0027	0.0029	0.0031	0.0036
		±0.5	**0.0455**	0.0451	0.0454	0.0457	0.0462	**0.0027**	0.0027	0.0030	0.0032	0.0036
2	7	0	**0.0187**	0.0187	0.0193	0.0199	0.0212	**0.0004**	0.0027	0.0028	0.0029	0.0032
		±0.5	**0.0187**	0.0182	0.0189	0.0195	0.0208	**0.0004**	0.0028	0.0029	0.0030	0.0032
	10	0	**0.0259**	0.0259	0.0264	0.0269	0.0279	**0.0008**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0259**	0.0255	0.0260	0.0265	0.0274	**0.0008**	0.0028	0.0029	0.0031	0.0033
	15	0	**0.0321**	0.0321	0.0324	0.0327	0.0334	**0.0013**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0321**	0.0317	0.0321	0.0324	0.0331	**0.0013**	0.0028	0.0029	0.0030	0.0033
4	7	0	**0.0027**	0.0027	0.0029	0.0032	0.0036	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0027**	0.0028	0.0030	0.0032	0.0036	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0106**	0.0106	0.0109	0.0113	0.0120	**0.0001**	0.0027	0.0027	0.0028	0.0028
		±0.5	**0.0106**	0.0104	0.0108	0.0111	0.0118	**0.0001**	0.0027	0.0028	0.0028	0.0029
	15	0	**0.0202**	0.0202	0.0205	0.0208	0.0214	**0.0005**	0.0027	0.0028	0.0028	0.0029
		±0.5	**0.0202**	0.0200	0.0203	0.0206	0.0212	**0.0005**	0.0027	0.0028	0.0029	0.0030
6	7	0	**0.0000**	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0000**	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0020**	0.0020	0.0021	0.0022	0.0024	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0020**	0.0020	0.0021	0.0022	0.0025	**0.0000**	0.0027	0.0027	0.0027	0.0027
	15	0	**0.0106**	0.0106	0.0108	0.0110	0.0115	**0.0001**	0.0027	0.0027	0.0027	0.0028
		±0.5	**0.0106**	0.0105	0.0107	0.0110	0.0114	**0.0001**	0.0027	0.0027	0.0028	0.0028

Continued…

r=6 ρ=0.9864												
v			k=2					k=3				
	n	λ_4 λ_3	**α**	0	0.5	1	2	**α**	0	0.5	1	2
0	7	0	**0.0455**	0.0455	0.0461	0.0468	0.0481	**0.0027**	0.0027	0.0032	0.0036	0.0046
		±0.5	**0.0455**	0.0445	0.0452	0.0458	0.0471	**0.0027**	0.0028	0.0033	0.0037	0.0047
	10	0	**0.0455**	0.0455	0.0460	0.0464	0.0473	**0.0027**	0.0027	0.0030	0.0034	0.0040
		±0.5	**0.0455**	0.0448	0.0453	0.0457	0.0466	**0.0027**	0.0028	0.0031	0.0034	0.0041
	15	0	**0.0455**	0.0455	0.0458	0.0461	0.0467	**0.0027**	0.0027	0.0029	0.0031	0.0036
		±0.5	**0.0455**	0.0451	0.0454	0.0457	0.0462	**0.0027**	0.0027	0.0030	0.0032	0.0036
2	7	0	**0.0183**	0.0183	0.0189	0.0196	0.0208	**0.0004**	0.0027	0.0028	0.0029	0.0031
		±0.5	**0.0183**	0.0179	0.0185	0.0192	0.0204	**0.0004**	0.0028	0.0029	0.0030	0.0032
	10	0	**0.0256**	0.0256	0.0261	0.0266	0.0276	**0.0008**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0256**	0.0252	0.0257	0.0262	0.0271	**0.0008**	0.0028	0.0029	0.0030	0.0033
	15	0	**0.0319**	0.0319	0.0322	0.0325	0.0332	**0.0013**	0.0027	0.0028	0.0030	0.0032
		±0.5	**0.0319**	0.0315	0.0319	0.0322	0.0329	**0.0013**	0.0028	0.0029	0.0030	0.0033
4	7	0	**0.0025**	0.0025	0.0027	0.0029	0.0033	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0025**	0.0025	0.0027	0.0029	0.0033	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0102**	0.0102	0.0105	0.0108	0.0115	**0.0001**	0.0027	0.0027	0.0028	0.0028
		±0.5	**0.0102**	0.0100	0.0104	0.0107	0.0114	**0.0001**	0.0027	0.0028	0.0028	0.0028
	15	0	**0.0198**	0.0198	0.0201	0.0204	0.0210	**0.0005**	0.0027	0.0028	0.0028	0.0029
		±0.5	**0.0198**	0.0196	0.0199	0.0202	0.0208	**0.0005**	0.0027	0.0028	0.0029	0.0030
6	7	0	**0.0000**	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0000**	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0027	0.0027	0.0027	0.0027
	10	0	**0.0017**	0.0017	0.0019	0.0020	0.0022	**0.0000**	0.0027	0.0027	0.0027	0.0027
		±0.5	**0.0017**	0.0018	0.0019	0.0020	0.0022	**0.0000**	0.0027	0.0027	0.0027	0.0027
	15	0	**0.0102**	0.0102	0.0104	0.0106	0.0111	**0.0001**	0.0027	0.0027	0.0027	0.0028
		±0.5	**0.0102**	0.0101	0.0103	0.0105	0.0110	**0.0001**	0.0027	0.0027	0.0028	0.0028

Table-2: OC Function of Control Charts for Mean under Measurement Error with Known cv

v	r=∞, ρ=1, k=3, n=7								
	$\lambda_3\lambda_4$→ γ↓	(0,0)	(0,0.5)	(0,1.0)	(0,2.0)	(0.5,0)	(0.5,0.5)	(0.5,1.0)	(0.5,2.0)
0	-3.5	0.3085	0.3071	0.3057	0.3028	0.3013	0.2999	0.2984	0.2956
	-3	0.5000	0.5000	0.5000	0.5000	0.4874	0.4874	0.4874	0.4874
	-2.5	0.6915	0.6929	0.6943	0.6972	0.6821	0.6835	0.6849	0.6878
	-2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	-1	0.9772	0.9769	0.9765	0.9759	0.9827	0.9824	0.9820	0.9813
	0	0.9973	0.9968	0.9964	0.9954	0.9972	0.9967	0.9963	0.9953
	1	0.9772	0.9769	0.9765	0.9759	0.9726	0.9723	0.9719	0.9713
	2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	2.5	0.6915	0.6929	0.6943	0.6972	0.6987	0.7001	0.7016	0.7044
	3	0.5000	0.5000	0.5000	0.5000	0.5126	0.5126	0.5126	0.5126
	3.5	0.3085	0.3071	0.3057	0.3028	0.3179	0.3165	0.3151	0.3122
2	-3.5	0.5198	0.5199	0.5201	0.5203	0.5091	0.5092	0.5094	0.5096
	-3	0.7087	0.7098	0.7109	0.7130	0.7015	0.7026	0.7037	0.7059
	-2.5	0.8531	0.8540	0.8550	0.8570	0.8532	0.8542	0.8552	0.8571
	-2	0.9394	0.9396	0.9399	0.9403	0.9441	0.9443	0.9446	0.9450
	-1	0.9946	0.9943	0.9940	0.9934	0.9970	0.9967	0.9964	0.9958
	0	0.9996	0.9995	0.9994	0.9992	0.9995	0.9994	0.9993	0.9991
	1	0.9946	0.9943	0.9940	0.9934	0.9925	0.9922	0.9919	0.9913
	2	0.9394	0.9396	0.9399	0.9403	0.9351	0.9354	0.9356	0.9361
	2.5	0.8531	0.8540	0.8550	0.8570	0.8520	0.8530	0.8539	0.8559
	3	0.7087	0.7098	0.7109	0.7130	0.7143	0.7154	0.7164	0.7186
	3.5	0.5198	0.5199	0.5201	0.5203	0.5303	0.5304	0.5305	0.5308
4	-3.5	0.8605	0.8611	0.8616	0.8627	0.8611	0.8616	0.8622	0.8633
	-3	0.9432	0.9434	0.9435	0.9437	0.9469	0.9470	0.9472	0.9474
	-2.5	0.9814	0.9812	0.9810	0.9807	0.9847	0.9845	0.9844	0.9840
	-2	0.9951	0.9949	0.9948	0.9944	0.9968	0.9966	0.9965	0.9961
	-1	0.9998	0.9998	0.9998	0.9997	1.0000	0.9999	0.9999	0.9998
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.9998	0.9998	0.9998	0.9997	0.9996	0.9996	0.9996	0.9995
	2	0.9951	0.9949	0.9948	0.9944	0.9935	0.9933	0.9931	0.9928
	2.5	0.9814	0.9812	0.9810	0.9807	0.9784	0.9782	0.9781	0.9778
	3	0.9432	0.9434	0.9435	0.9437	0.9398	0.9400	0.9401	0.9403
	3.5	0.8605	0.8611	0.8616	0.8627	0.8595	0.8600	0.8606	0.8617
6	-3.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Continued…

v	r=2, ρ=0.8944, k=3, n=7,								
	$\lambda_3\lambda_4$→ γ↓	(0,0)	(0,0.5)	(0,1.0)	(0,2.0)	(0.5,0)	(0.5,0.5)	(0.5,1.0)	(0.5,2.0)
0	-3.5	0.3085	0.3071	0.3057	0.3028	0.3013	0.2999	0.2984	0.2956
	-3	0.5000	0.5000	0.5000	0.5000	0.4874	0.4874	0.4874	0.4874
	-2.5	0.6915	0.6929	0.6943	0.6972	0.6821	0.6835	0.6849	0.6878
	-2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	-1	0.9772	0.9769	0.9765	0.9759	0.9827	0.9824	0.9820	0.9813
	0	0.9973	0.9968	0.9964	0.9954	0.9972	0.9967	0.9963	0.9953
	1	0.9772	0.9769	0.9765	0.9759	0.9726	0.9723	0.9719	0.9713
	2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	2.5	0.6915	0.6929	0.6943	0.6972	0.6987	0.7001	0.7016	0.7044
	3	0.5000	0.5000	0.5000	0.5000	0.5126	0.5126	0.5126	0.5126
	3.5	0.3085	0.3071	0.3057	0.3028	0.3179	0.3165	0.3151	0.3122
2	-3.5	0.4908	0.4908	0.4907	0.4906	0.4800	0.4800	0.4799	0.4798
	-3	0.6833	0.6844	0.6854	0.6875	0.6750	0.6761	0.6771	0.6792
	-2.5	0.8357	0.8368	0.8379	0.8401	0.8348	0.8359	0.8370	0.8392
	-2	0.9302	0.9305	0.9309	0.9316	0.9346	0.9350	0.9353	0.9360
	-1	0.9934	0.9930	0.9927	0.9921	0.9961	0.9958	0.9954	0.9948
	0	0.9995	0.9994	0.9992	0.9990	0.9994	0.9993	0.9991	0.9989
	1	0.9934	0.9930	0.9927	0.9921	0.9909	0.9906	0.9903	0.9896
	2	0.9302	0.9305	0.9309	0.9316	0.9260	0.9264	0.9267	0.9274
	2.5	0.8357	0.8368	0.8379	0.8401	0.8354	0.8365	0.8376	0.8398
	3	0.6833	0.6844	0.6854	0.6875	0.6900	0.6910	0.6921	0.6942
	3.5	0.4908	0.4908	0.4907	0.4906	0.5017	0.5016	0.5016	0.5015
4	-3.5	0.7854	0.7862	0.7870	0.7886	0.7825	0.7832	0.7840	0.7856
	-3	0.9016	0.9020	0.9024	0.9033	0.9040	0.9045	0.9049	0.9058
	-2.5	0.9633	0.9633	0.9632	0.9631	0.9674	0.9674	0.9674	0.9673
	-2	0.9890	0.9888	0.9886	0.9881	0.9919	0.9917	0.9914	0.9910
	-1	0.9995	0.9994	0.9994	0.9992	0.9999	0.9998	0.9997	0.9996
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	1	0.9995	0.9994	0.9994	0.9992	0.9991	0.9990	0.9989	0.9988
	2	0.9890	0.9888	0.9886	0.9881	0.9865	0.9862	0.9860	0.9856
	2.5	0.9633	0.9633	0.9632	0.9631	0.9596	0.9596	0.9596	0.9595
	3	0.9016	0.9020	0.9024	0.9033	0.8990	0.8994	0.8998	0.9007
	3.5	0.7854	0.7862	0.7870	0.7886	0.7873	0.7881	0.7889	0.7905
6	-3.5	0.9966	0.9966	0.9965	0.9963	0.9976	0.9975	0.9975	0.9973
	-3	0.9993	0.9993	0.9993	0.9992	0.9996	0.9996	0.9996	0.9995
	-2.5	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	0.9999
	-2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.5	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998
	3	0.9993	0.9993	0.9993	0.9992	0.9990	0.9990	0.9989	0.9988
	3.5	0.9966	0.9966	0.9965	0.9963	0.9957	0.9956	0.9955	0.9953

Continued…

v	**r=4, ρ=0.9701, k=3, n=7**								
	$\lambda_3\lambda_4$→ γ↓	(0,0)	(0,0.5)	(0,1.0)	(0,2.0)	(0.5,0)	(0.5,0.5)	(0.5,1.0)	(0.5,2.0)
0	-3.5	0.3085	0.3071	0.3057	0.3028	0.3013	0.2999	0.2984	0.2956
	-3	0.5000	0.5000	0.5000	0.5000	0.4874	0.4874	0.4874	0.4874
	-2.5	0.6915	0.6929	0.6943	0.6972	0.6821	0.6835	0.6849	0.6878
	-2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	-1	0.9772	0.9769	0.9765	0.9759	0.9827	0.9824	0.9820	0.9813
	0	0.9973	0.9968	0.9964	0.9954	0.9972	0.9967	0.9963	0.9953
	1	0.9772	0.9769	0.9765	0.9759	0.9726	0.9723	0.9719	0.9713
	2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	2.5	0.6915	0.6929	0.6943	0.6972	0.6987	0.7001	0.7016	0.7044
	3	0.5000	0.5000	0.5000	0.5000	0.5126	0.5126	0.5126	0.5126
	3.5	0.3085	0.3071	0.3057	0.3028	0.3179	0.3165	0.3151	0.3122
2	-3.5	0.5114	0.5115	0.5116	0.5117	0.5007	0.5008	0.5008	0.5010
	-3	0.7015	0.7025	0.7036	0.7058	0.6940	0.6950	0.6961	0.6983
	-2.5	0.8482	0.8492	0.8502	0.8522	0.8481	0.8491	0.8501	0.8521
	-2	0.9368	0.9371	0.9374	0.9379	0.9415	0.9417	0.9420	0.9425
	-1	0.9943	0.9940	0.9937	0.9931	0.9967	0.9964	0.9961	0.9955
	0	0.9996	0.9995	0.9994	0.9991	0.9995	0.9994	0.9993	0.9990
	1	0.9943	0.9940	0.9937	0.9931	0.9920	0.9917	0.9914	0.9908
	2	0.9368	0.9371	0.9374	0.9379	0.9326	0.9328	0.9331	0.9337
	2.5	0.8482	0.8492	0.8502	0.8522	0.8473	0.8483	0.8493	0.8514
	3	0.7015	0.7025	0.7036	0.7058	0.7073	0.7084	0.7095	0.7116
	3.5	0.5114	0.5115	0.5116	0.5117	0.5220	0.5221	0.5222	0.5223
4	-3.5	0.8399	0.8405	0.8411	0.8424	0.8395	0.8401	0.8408	0.8421
	-3	0.9324	0.9326	0.9328	0.9332	0.9359	0.9361	0.9363	0.9367
	-2.5	0.9769	0.9768	0.9766	0.9764	0.9806	0.9804	0.9803	0.9800
	-2	0.9937	0.9935	0.9933	0.9929	0.9957	0.9955	0.9953	0.9950
	-1	0.9998	0.9997	0.9997	0.9996	0.9999	0.9999	0.9999	0.9998
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.9998	0.9997	0.9997	0.9996	0.9995	0.9995	0.9994	0.9994
	2	0.9937	0.9935	0.9933	0.9929	0.9918	0.9916	0.9914	0.9911
	2.5	0.9769	0.9768	0.9766	0.9764	0.9737	0.9736	0.9734	0.9731
	3	0.9324	0.9326	0.9328	0.9332	0.9291	0.9293	0.9295	0.9299
	3.5	0.8399	0.8405	0.8411	0.8424	0.8396	0.8402	0.8409	0.8422
6	-3.5	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999
	-3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998

Continued…

v	**r=6, ρ=0.9864, k=3, n=7**								
	$\lambda_3\lambda_4$→ γ↓	(0,0)	(0,0.5)	(0,1.0)	(0,2.0)	(0.5,0)	(0.5,0.5)	(0.5,1.0)	(0.5,2.0)
0	-3.5	0.3085	0.3071	0.3057	0.3028	0.3013	0.2999	0.2984	0.2956
	-3	0.5000	0.5000	0.5000	0.5000	0.4874	0.4874	0.4874	0.4874
	-2.5	0.6915	0.6929	0.6943	0.6972	0.6821	0.6835	0.6849	0.6878
	-2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	-1	0.9772	0.9769	0.9765	0.9759	0.9827	0.9824	0.9820	0.9813
	0	0.9973	0.9968	0.9964	0.9954	0.9972	0.9967	0.9963	0.9953
	1	0.9772	0.9769	0.9765	0.9759	0.9726	0.9723	0.9719	0.9713
	2	0.8413	0.8428	0.8442	0.8471	0.8406	0.8421	0.8435	0.8464
	2.5	0.6915	0.6929	0.6943	0.6972	0.6987	0.7001	0.7016	0.7044
	3	0.5000	0.5000	0.5000	0.5000	0.5126	0.5126	0.5126	0.5126
	3.5	0.3085	0.3071	0.3057	0.3028	0.3179	0.3165	0.3151	0.3122
2	-3.5	0.5160	0.5161	0.5162	0.5164	0.5053	0.5054	0.5055	0.5057
	-3	0.7054	0.7065	0.7076	0.7097	0.6981	0.6992	0.7002	0.7024
	-2.5	0.8508	0.8518	0.8528	0.8548	0.8509	0.8519	0.8529	0.8548
	-2	0.9382	0.9385	0.9387	0.9392	0.9429	0.9431	0.9434	0.9439
	-1	0.9945	0.9942	0.9939	0.9933	0.9969	0.9966	0.9963	0.9957
	0	0.9996	0.9995	0.9994	0.9992	0.9995	0.9994	0.9993	0.9991
	1	0.9945	0.9942	0.9939	0.9933	0.9923	0.9920	0.9917	0.9911
	2	0.9382	0.9385	0.9387	0.9392	0.9340	0.9342	0.9345	0.9350
	2.5	0.8508	0.8518	0.8528	0.8548	0.8499	0.8509	0.8519	0.8538
	3	0.7054	0.7065	0.7076	0.7097	0.7111	0.7122	0.7133	0.7154
	3.5	0.5160	0.5161	0.5162	0.5164	0.5265	0.5266	0.5267	0.5269
4	-3.5	0.8512	0.8518	0.8524	0.8536	0.8513	0.8519	0.8525	0.8537
	-3	0.9384	0.9386	0.9387	0.9390	0.9420	0.9422	0.9423	0.9426
	-2.5	0.9794	0.9792	0.9791	0.9788	0.9829	0.9827	0.9826	0.9823
	-2	0.9945	0.9943	0.9941	0.9938	0.9963	0.9962	0.9960	0.9956
	-1	0.9998	0.9998	0.9997	0.9997	1.0000	0.9999	0.9999	0.9998
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.9998	0.9998	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	2	0.9945	0.9943	0.9941	0.9938	0.9928	0.9926	0.9924	0.9920
	2.5	0.9794	0.9792	0.9791	0.9788	0.9763	0.9762	0.9760	0.9757
	3	0.9384	0.9386	0.9387	0.9390	0.9351	0.9352	0.9354	0.9357
	3.5	0.8512	0.8518	0.8524	0.8536	0.8505	0.8511	0.8517	0.8529
6	-3.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

It was found that these errors and non-normality parameters affects the performance of the OC function. Therefore, we suggest a multiple measurement proved to be a solution to this problem. However, the extra money and time need is another problem.

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